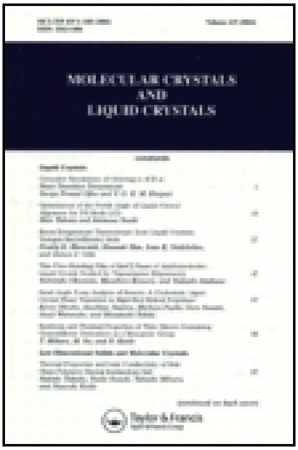

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Numerical Simulation of Optical Parameters of LCD with Dichroic Nematic Liquid Crystal

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Numerical Simulation of Optical Parameters of LCD with Dichroic Nematic Liquid Crystal

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The aim of the present paper is to show the mathematical model of light propagation through the liquid crystals display. The numerical programme based on this model is constructed. The possibilities of this programme is shown.

Keywords: liquid crystals display; numerical modelling; optimization of optical parameters

INTRODUCTION

Capability of optimization of different types of display is very important from the proper electrooptical effect chose point of view. Very important seen: to be conclusion about employing the optimum components and technology to reach the expected results.

Nevertheless, even if there is worked out a basic optimization procedure, obtaining detailed data concerning the optimum matching of display's components requires a lot of experimental work over determining (sometimes very frequent), the properties of a chosen set of examined elements. Thus, the role of computer modelling of display's work, taking into account all the above reservation is a helpful, very often the necessary instrument for technologists.

This paper presents considerations over light wave propagation through LC display employing nematic LC (also dichroic) and working under conditions approximating real ones, These enable us to make numerical programme modelling operation of LC display. The following factors are taken into account:

- real spectral properties of polarizing films;

- spectral characteristics of light source;
- dispersion of compound refraction indexes for particular elements of a display;
- extinction coefficients of individual components of a display;
- polarizer's orientation, tilt and texture angle;
- dichroic properties of LC layer;
- properties of external source of light;
- human eyes spectral sensitivity for different conditions of observer.

MATHEMATICAL RUDIMENTS

Mathematical model of LC transducer's operation employ GOA method (Geometry Optics Approximation)[1,2,3,4,5,6,7,8] including influence of light phase changes while passing and reflecting from phase boundaries (at least one of them has absorptive properties) and taking into account interference and dichroic properties. Real transmission characteristics of polarizers and conductive layers are considered. Basic scheme of beam of light propagating through display's structure is presented in Fig. 1. In the same way reflective mode can be performed. Fig. 2 defies coordinates system we have assumed. As one can see this is a typical geometric system taken up in accordance with GOA method.

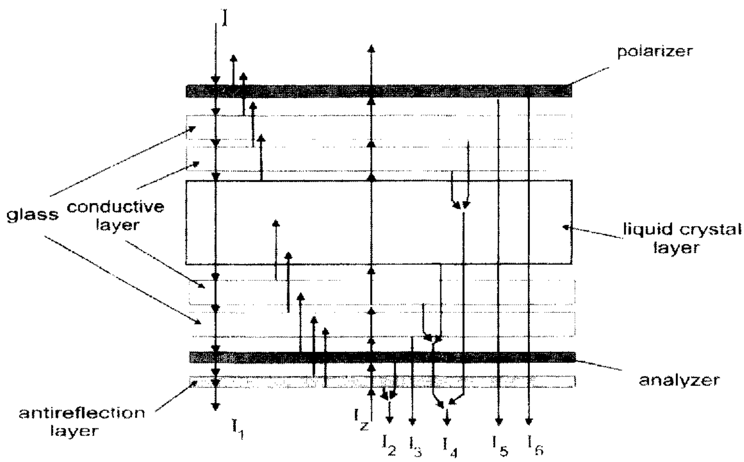

FIGURE 1 Passage of a beam of light through a structure of LC display working in transmission state.

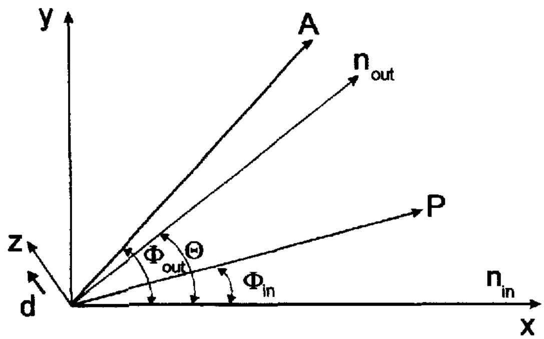

FIGURE 2 Geometry of LC transducer we have considered in calculations. Cell's thickness changes according to z axis.

n_{in} - inward director's orientation (an average directions of positions of long axes of LC molecules) at the input plate overlaps with x axis of the assumed coordinates system.

n_{out} - outward director's orientation changes by an angle Θ (texture twist angle – for TN effect Θ=90^o);

P - polarizer's axis forms an angle Φ_{in} with x axis;

A - analyzer's axis forms an angle Φ_{out} with x axis.

The point of issue of our calculations is introduction of another coordinates x'y' (different from the xy system in Fig. 2) connected with indicatrix of reflection indexes projection on the plane located in parallel to the LC layer. We have assumed, that the long axis projection is oriented in accordant with x' axis direction. At the layer's boundary on the input polarizer's side the x'y' system of coordinates overlaps with the xy system.

Intensity of the incident scattered light is changed by a single polarizer (regardless of reflections) in the following way:

$$I_{\|} = \frac{1}{2} T_p(\|) \cdot I_0 \qquad (1)$$

$$I_{+} = \frac{1}{2} T_p(+) \cdot I_0 \qquad (2)$$

where: $I_{\|}$ and $T_p(\|)$ are respectively: output intensity and polarizer's transmission for the light polarized linearly along it's optical axis, I_+ and $T_p(+)$ – for the light polarized linearly perpendicularly to the optical axis. I_0 denotes intensity of incident light.

Developing equations (1) and (2) and taking into account refractive index for a polarizer R_{pp} we get a transmission formula for two polarizers oriented in parallel and perpendicularly ($T_{pm}^{\parallel}$ and T_{pm}^{+}), from which it is easily to determine $T_p(\parallel)$ and $T_p(+)$:

$$\tfrac{1}{2}\cdot\left(1-R_{pp}\right)^2\cdot\left[T_p^2(\parallel)+T_p^2(+)\right]=T_{pm}^{\parallel} \tag{3}$$

$$T_p(\parallel)\cdot T_p(+)\cdot\left(1-R_{pp}\right)^2=T_{pm}^{+} \tag{4}$$

Then we can start the analysis of light propagation through the structure of LC display presented in Fig. 1. Let's define transmission coefficients $t_{1\text{-}2}$ and $t_{2\text{-}1}$ and refraction coefficients $r_{1\text{-}2}$ and $r_{2\text{-}1}$ for an amplitude of vector E of the light for its transition from the medium 1 to 2, then from 2 to 1 as:

$$\begin{aligned} t_{1-2}&=\frac{2\cdot\hat{n}_1}{\hat{n}_1+\hat{n}_2}; & t_{2-1}&=\frac{2\cdot\hat{n}_2}{\hat{n}_1+\hat{n}_2};\\ r_{1-2}&=\frac{\hat{n}_1-\hat{n}_2}{\hat{n}_1+\hat{n}_2}; & r_{2-1}&=\frac{\hat{n}_2-\hat{n}_1}{\hat{n}_2+\hat{n}_1}; \end{aligned} \tag{5}$$

where $\hat{n}$ is an absolute compound refractive index value.
Let's present vector E as: $E=A\cdot e^{i\cdot(\omega t-K\cdot z_0)}$ (where ω – periodicity of the wave, K – wave vector in the air, z_o – optical distance covered by the wave along the z axis (Fig. 2) and z_o=0) with normalized amplitude A=1. At the point of light entrance into LC layer we obtain (for parallel and perpendicular positions of polarizer) the following formulae:

$$E^{P}{}_{\parallel}=t_{in}\cdot\sqrt{\frac{1}{2}T^{P}(\parallel)\cdot e^{-\alpha_{ITO}\cdot d_{ITO}}}\cdot e^{i\cdot(\omega t-\phi_{in})} \tag{6}$$

$$E^{P}{}_{+}=t_{in}\cdot\sqrt{\frac{1}{2}T^{P}(+)\cdot e^{-\alpha_{ITO}\cdot d_{ITO}}}\cdot e^{i\cdot(\omega t-\phi_{in})} \tag{7}$$

Index p is reserved for the light crossing an input layers – a polarizer and remaining layers positioned between LC layer and the source of light but not yet conductive layer-LC border; t_{in} denotes transmission of an amplitude of vector E through borders of input layers, Φ_{in} is phase shift value resulting from covering the distance of input layers (($\phi_{in}=\sum_i K_i\cdot z_i$ where K_i is wave vector in medium i and z_i is thickness of the medium i). d_{ITO} is thickness of the conductive layer. α_{ITO} stands for absorption coefficient for a conductive layer. It may be obtained by the analysis of light transmission through a set of glass plate-conductive layer (e.g. ITO) which leads to the equation:

$$\frac{A\cdot\left(B+\alpha_{ITO}^2\right)\cdot e^{-\alpha_{ITO}\cdot d_{ITO}}}{\left(C+\alpha_{ITO}^2\right)\cdot\left(D+\alpha_{ITO}^2\right)}\cdot\left\{\begin{array}{l} 1+e^{-2\cdot\alpha_{ITO}\cdot d_{ITO}}\cdot\left[\frac{\left(E+\alpha_{ITO}^2\right)\cdot\left(F+\alpha_{ITO}^2\right)}{\left(C+\alpha_{ITO}^2\right)\cdot\left(D+\alpha_{ITO}^2\right)}\right]^2+ \\ +2\cdot e^{-\alpha_{ITO}\cdot d_{ITO}}\cdot\frac{\left(E+\alpha_{ITO}^2\right)\cdot\left(F+\alpha_{ITO}^2\right)}{\left(C+\alpha_{ITO}^2\right)\cdot\left(D+\alpha_{ITO}^2\right)}\cdot\cos\gamma \end{array}\right\}-T_{ITOm}=0 \tag{8}$$

where:

$$A=\left[\frac{32\cdot\pi\cdot n_g}{\lambda\cdot\left(n_g+1\right)}\right]^2;\quad B=\left[\frac{4\cdot\pi\cdot n_{ITO}}{\lambda}\right]^2;\quad C=\left[\frac{4\cdot\pi\cdot\left(n_{ITO}+n_g\right)}{\lambda}\right]^2;$$
$$D=\left[\frac{4\cdot\pi\cdot\left(n_{ITO}+1\right)}{\lambda}\right]^2;\quad E=\left[\frac{4\cdot\pi\cdot\left(n_{ITO}-n_g\right)}{\lambda}\right]^2;\quad F=\left[\frac{4\cdot\pi\cdot\left(n_{ITO}-1\right)}{\lambda}\right]^2; \tag{9}$$

and:

$$\begin{aligned}\gamma = arc\,\mathrm{tg}&\left(\frac{-8\cdot\pi\cdot\alpha_{ITO}}{\left(\lambda\cdot n_{ITO}\right)^2+4\cdot\pi\cdot\alpha_{ITO}^2-\lambda^2}\right)+\\ +arc\,\mathrm{tg}&\left(\frac{-8\cdot\pi\cdot\alpha_{ITO}\cdot n_g}{\left(\lambda\cdot n_{ITO}\right)^2+4\cdot\pi\cdot\alpha_{ITO}^2-\lambda^2\cdot n_g}\right)-\frac{4\cdot\pi\cdot n_{ITO}\cdot d_{ITO}}{\lambda}\end{aligned} \tag{10}$$

n_{ITO} and n_g are the refractive indexes of conductive layer and glass, respectively.

We can solve the equation applying the "falsi" (chords) method. For given refractive coefficient n_g (for glass) and n_{ITO} (for conductive layer) and conductive layer thickness d_{ITO} one should take an initial value of $\alpha_{ITO}=0$ and increasing it's value according to the "falsi" method, then search (with a given accuracy) a value equal zero of the function positioned on the left side of equation mark in expression (8). The discussed method is not only effective approach to determining the real value of absorption coefficient of the conductive layer in the whole visual range, but controlling experimental data correctness as well. If (for experimentally obtained values of refractive indexes, layer's thickness and glass-polarizing film transmission) the left side of equation (8) gives negative value this indicates an error in measurements of one of the presented parameters.

These vectors of light (6) and (7) have to be discussed separately due to lack of phase correlation between them in the scattered light. In other words, we are not in position to settle their senses.

From the $E^{p}{}_{||}$, formula (6) according with Fig. 2 it appears, that we can determine intensities of vector E, alongside the x (director) and y (perpendicularly to the director):

$$E^{p}(x) = B^{p} \cdot e^{i \cdot (\omega t - \phi_{in})} \tag{11}$$

$$E^{p}(y) = D^{p} \cdot e^{i \cdot (\omega t - \phi_{in})} \tag{12}$$

where:

$$\begin{aligned} B^{p} &= t_{in} \cdot \sqrt{\frac{1}{2} T_{p}(\|) \cdot e^{-\alpha_{ITO} \cdot d_{ITO}}} \cdot \cos \Phi_{in} \\ D^{p} &= t_{in} \cdot \sqrt{\frac{1}{2} T_{p}(\|) \cdot e^{-\alpha_{ITO} \cdot d_{ITO}}} \cdot \sin \Phi_{in} \end{aligned} \tag{13}$$

At the point on entrance into LC layer the following condition is fulfilled:

$$E^{0}(x) = t_{ITO-neff} \cdot E^{p}(x) \text{ and } E^{0}(y) = t_{ITO-no} \cdot E^{p}(y) \tag{14}$$

where $E^{0}(x)$ and $E^{0}(y)$ denote initial values of intensities of vector E in LC layer for extraordinary and ordinary rays, respectively. $t_{ITO-neff}$ and t_{ITO-no} are transmission coefficients for light amplitude while crossing the border of conductive layer-LC for polarization parallel and perpendicular to optical axis of LC. n_{eff} denotes the value of effective refraction index resulting from the tilt of director towards wave plane by the angle Θ_s and it's value can be determined by analysis of ellipsoid of resolution presenting an indicatrix of refractive indexes.

$$n_{eff} = n_{e} \cdot n_{o} \cdot \sqrt{\frac{1 + \mathrm{tg}^{2} \Theta_{s}}{n_{o}^{2} + n_{e}^{2} \cdot \mathrm{tg}^{2} \Theta_{s}}}\,. \tag{15}$$

It is easy to prove, that (as it is in the polarizer's case) for a layer of LC containing dichroic dye, the values of extinction coefficient are much smaller than real part of it's refractive index. Thus we can assume, that LC layer possess only real value of refractive index. t_{ITO-ne} and t_{ITO-no} can be determined (in the same way we can determine t_{g-ITO}):

$$\begin{aligned} t_{ITO-neff} &= \frac{2 \cdot (n_{ITO} - i \cdot k_{ITO})}{n_{eff} + n_{ITO} - i \cdot k_{ITO}} = \left| t_{ITO-neff} \right| \cdot e^{i \cdot \phi_{ITO-neff}} \\ t_{ITO-no} &= \frac{2 \cdot (n_{ITO} - i \cdot k_{ITO})}{n_{o} + n_{ITO} - i \cdot k_{ITO}} = \left| t_{ITO-no} \right| \cdot e^{i \cdot \phi_{ITO-no}} \end{aligned} \tag{16}$$

where:

$$\left|t_{ITO-neff}\right| = 2\cdot\sqrt{\frac{n_{ITO}^2+k_{ITO}^2}{\left(n_{eff}+n_{ITO}\right)^2+k_{ITO}^2}}\ \ and\ \ \phi_{ITO-neff} = arc\,tg\left(\frac{-k_{ITO}\cdot n_{eff}}{n_{ITO}\cdot\left(n_{eff}+n_{ITO}\right)+k_{ITO}^2}\right) \quad (17)$$

$$\left|t_{ITO-no}\right| = 2\cdot\sqrt{\frac{n_{ITO}^2+k_{ITO}^2}{\left(n_o+n_{ITO}\right)^2+k_{ITO}^2}}\ \ and\ \ \phi_{ITO-no} = arc\,tg\left(\frac{-k_{ITO}\cdot n_o}{n_{ITO}\cdot\left(n_o+n_{ITO}\right)+k_{ITO}^2}\right)$$

Values of vectors (14) at any point of LC layer describe polarization properties of the light propagating across a display. Further description of light propagation through a LC transducer will be performed for real part of vector E in x'y' coordinates system rotating together with a director of LC layer. It is necessary for simplifying vector's notation. This, in turn, makes it possible to determine amplitude values at any point of (in general indefinite) number of LC layers. If δz denotes thickness of a layer, δΘ – director's angle of rotation, then values of intensity vectors within x'y' coordinates system before passing the first of such a layer of a linearly birefringence medium can be put as:

$$E^0(x') = E^0(x)\cdot\cos\delta\Theta + E^0(y)\cdot\sin\delta\Theta \quad (15)$$

$$E^0(y') = -E^0(x)\cdot\sin\delta\Theta + E^0(y)\cdot\cos\delta\Theta \quad (16)$$

and after covering a distance equal to the layer's thickness due to difference of extraordinary rays' velocities and under assumption, that $\delta\Theta \to 0$:

$$E^1(x') = \left|t_{ITO-neff}\right|\cdot\left(\left|B^p\right|\cdot\cos\delta\Theta + \left|D^p\right|\cdot\sin\delta\Theta\right)\cdot\cos\left(\omega t - \phi_{in} + \phi_{ITO-neff} - \delta_e\right) \quad (17)$$

$$E^1(y') = \left|t_{ITO-no}\right|\cdot\left(-\left|B^p\right|\cdot\sin\delta\Theta + \left|D^p\right|\cdot\cos\delta\Theta\right)\cdot\cos\left(\omega t - \phi_{in} + \phi_{ITO-no} - \delta_o\right) \quad (18)$$

and also:

$$E^1(x') = Ampx'\cdot\cos\left(\omega t + \delta_w\right) \quad (19)$$

$$E^1(y') = \pm Ampy'\cdot\cos\left(\omega t + 90^0 + \delta_w\right) \quad (20)$$

where: Ampx' and Ampy' are amplitudes of vector E along x' and y' axes, δ_e and δ_o represent phase shifts occurring after passing a distance δz in LC medium. For extraordinary and ordinary rays of light they are defined as $\delta_e = \frac{2\pi n_{eff}\delta z}{\lambda}$ and $\delta_o = \frac{2\pi n_o \delta z}{\lambda}$, respectively. δ_w is a phase shift resulting from rotation of the coordinates system, index 1 denotes

values of vectors E after passing through the 1st layer and 0 before the passing.
According to the following rotation:

$$\begin{aligned}
A^1 &= -\left|t_{ITO-neff}\right| \cdot \left(\left|B^p\right| \cdot \cos\delta\Theta + \left|D^p\right| \cdot \sin\delta\Theta\right) \cdot \sin\left(\phi_{ITO-neff} - \phi_{wej} - \delta_e\right) \\
B^1 &= \left|t_{ITO-neff}\right| \cdot \left(\left|B^p\right| \cdot \cos\delta\Theta + \left|D^p\right| \cdot \sin\delta\Theta\right) \cdot \cos\left(\phi_{ITO-neff} - \phi_{wej} - \delta_e\right) \\
C^1 &= -\left|t_{ITO-no}\right| \cdot \left(-\left|B^p\right| \cdot \sin\delta\Theta + \left|D^p\right| \cdot \cos\delta\Theta\right) \cdot \sin\left(\phi_{ITO-no} - \phi_{wej} - \delta_o\right) \\
D^1 &= \left|t_{ITO-no}\right| \cdot \left(-\left|B^p\right| \cdot \sin\delta\Theta + \left|D^p\right| \cdot \cos\delta\Theta\right) \cdot \cos\left(\phi_{ITO-no} - \phi_{wej} - \delta_o\right)
\end{aligned} \tag{21}$$

and from development of relations (17) and (18) we obtain:

$$E^1(x') = A^1 \cdot \sin\omega t + B^1 \cdot \cos\omega t \tag{22}$$

$$E^1(y') = C^1 \cdot \sin\omega t + D^1 \cdot \cos\omega t \tag{23}$$

Taking into account equations (19) and (20) we have:

$$E^1(x') = -Ampx' \cdot \sin\delta_w \cdot \sin\omega t + Ampx' \cdot \cos\delta_w \cdot \cos\omega t \tag{24}$$

$$E^1(y') = \pm Ampy' \cdot \cos\delta_w \cdot \sin\omega t \pm Ampy' \cdot \sin\delta_w \cdot \cos\omega t \tag{25}$$

By comparing expressions before sinωt and cosωt in equations (22) and (24) also (23) and (25) we get the following system of equations:

$$\begin{aligned}
-Ampx' \cdot \sin\delta_w &= A^1 \\
Ampx' \cdot \cos\delta_w &= B^1 \\
\pm Ampy' \cdot \cos\delta_w &= C^1 \\
\pm Ampy' \cdot \sin\delta_w &= D^1
\end{aligned} \tag{26}$$

After raising to the square the both sides of these equations, (the 1st , the 2nd then the 3rd and 4th, respectively) we are able to determine the squared amplitudes of vector E (or light intensity) in the x'y' coordinates system:

$$Ampx'^2 = \left(A^1\right)^2 + \left(B^1\right)^2 \text{ oraz } Ampy'^2 = \left(C^1\right)^2 + \left(D^1\right)^2 \tag{27}$$

At any point of a layer vectors E take up a form:

$$E^n(x') = A^n \cdot \sin\omega t + B^n \cdot \cos\omega t \tag{28}$$

$$E^n(y') = C^n \cdot \sin\omega t + D^n \cdot \cos\omega t \tag{29}$$

In general (taking into account dichroism of a layer) there have to be met the following relations:

$$A^n = \sqrt{e^{-\alpha_{\parallel}\cdot\delta z}} \cdot \begin{bmatrix} \left(A^{n-1}\cdot\cos\delta\Theta + C^{n-1}\cdot\sin\delta\Theta\right)\cdot\cos\delta_e + \\ + \left(B^{n-1}\cdot\cos\delta\Theta + D^{n-1}\cdot\sin\delta\Theta\right)\cdot\sin\delta_e \end{bmatrix} \quad (30)$$

$$B^n = \sqrt{e^{-\alpha_{\parallel}\cdot\delta z}} \cdot \begin{bmatrix} -\left(A^{n-1}\cdot\cos\delta\Theta + C^{n-1}\cdot\sin\delta\Theta\right)\cdot\sin\delta_e + \\ + \left(B^{n-1}\cdot\cos\delta\Theta + D^{n-1}\cdot\sin\delta\Theta\right)\cdot\cos\delta_e \end{bmatrix} \quad (31)$$

$$C^n = \sqrt{e^{-\alpha_{+}\cdot\delta z}} \cdot \begin{bmatrix} \left(-A^{n-1}\cdot\sin\delta\Theta + C^{n-1}\cdot\cos\delta\Theta\right)\cdot\cos\delta_o - \\ + \left(B^{n-1}\cdot\sin\delta\Theta - D^{n-1}\cdot\cos\delta\Theta\right)\cdot\sin\delta_o \end{bmatrix} \quad (32)$$

$$D^n = \sqrt{e^{-\alpha_{+}\cdot\delta z}} \cdot \begin{bmatrix} \left(A^{n-1}\cdot\sin\delta\Theta - C^{n-1}\cdot\cos\delta\Theta\right)\cdot\sin\delta_o - \\ + \left(B^{n-1}\cdot\sin\delta\Theta - D^{n-1}\cdot\cos\delta\Theta\right)\cdot\cos\delta_o \end{bmatrix} \quad (33)$$

Absorption coefficients can be expressed as:

$$\alpha_{\parallel} = -\ln\left(\frac{T_{\parallel}^{b}}{d}\right) \text{ and } \alpha_{+} = -\ln\left(\frac{T_{+}^{b}}{d}\right) \quad (34)$$

where $T_{\parallel}^{b}$ denotes transmission of a dichroic layer, having thickness d, for a light polarized according to an average director's orientation, while T_{+}^{b} describes transmission of the light polarized perpendicularly, measured within the system of two-beam spectrophotometer, where in the reference beam there is put an identical, basic LC mixture. Cells are illuminated with linearly polarized light and LC is ordered planarity (no twist).

Coefficient $\alpha_{\parallel}$ is dependent on director's tilt Θs in a layer in the way resulting from ellipse equation:

$$\alpha_{\parallel} = \alpha'_{\parallel}\cdot\alpha_{+}\cdot\sqrt{\frac{1+\mathrm{tg}^2\Theta_s}{\alpha_{+}^{2}+\alpha'^{2}_{\parallel}\cdot\mathrm{tg}^2\Theta_s}} \quad (35)$$

where $\alpha'_{\parallel}$ denotes absorption coefficient along director's orientation. This means, that having a form of vector E entering a layer we are able to determine it's polarization in x'y' coordinates at each point by cutting it into sufficiently sublayers (numerical method is applied). When a light wave reaches the end of a layer, vector E(x') oscillates in parallel to it's director while vector E(y') oscillates perpendicularly. According to geometry presented in Fig. 2 we can find the form of vector E reaching the analyzer, taking into account the influence of conductive layer and glass plate. Both directions: parallel and perpendicular to polarizer's axis are considered by formulae (36) and (37).

$$E_{\parallel}^{w} = P \cdot \begin{pmatrix} \left|t_{neff-ITO}\right| \cdot \sqrt{\left(A^{w}\right)^{2} + \left(B^{w}\right)^{2}} \cdot e^{i\phi_{neff-ITO}} \cdot \cos\left(\Phi_{out} - \Theta\right) + \\ + \left|t_{no-ITO}\right| \cdot \sqrt{\left(C^{w}\right)^{2} + \left(D^{w}\right)^{2}} \cdot e^{i\phi_{no-ITO}} \cdot \sin\left(\Phi_{out} - \Theta\right) \end{pmatrix} \cdot e^{i(\omega t + \delta_{w} - \phi_{out})} \quad (36)$$

$$E_{+}^{w} = \pm P \cdot \begin{pmatrix} -\left|t_{neff-ITO}\right| \cdot \sqrt{\left(A^{w}\right)^{2} + \left(B^{w}\right)^{2}} \cdot e^{i\phi_{neff-ITO}} \cdot \sin\left(\Phi_{out} - \Theta\right) + \\ + \left|t_{no-ITO}\right| \cdot \sqrt{\left(C^{w}\right)^{2} + \left(D^{w}\right)^{2}} \cdot e^{i\phi_{no-ITO}} \cdot \cos\left(\Phi_{out} - \Theta\right) \end{pmatrix} \cdot e^{i\left(\omega t + 90^{0} + \delta_{w} - \phi_{out}\right)} \quad (37)$$

where A^w, B^w, C^w and D^w are amplitudes of the light intensity vector oscillating in x' and y' while leaving a LC layer, $P = t_{ITO-sz} \cdot e^{-\alpha_{ITO} \cdot d_{ITO}}$, $\Phi_{neff-ITO}$ and Φ_{no-ITO} are amplitudes transmission and phase shifts for vectors passing a LC layer and entering conductive surface (for polarisation parallel and perpendicular to the optical axis, respectively). Φ_{out} represents here a phase change arising due to passing a conductive layer and plate of glass structure.

Finally, after coming out of an analyzer vector E take following mathematical shape:

$$E_{\parallel}^{a} = t_{out} \cdot \sqrt{T_a(\parallel)} \cdot E_{\parallel}^{w} \cdot e^{-i \cdot \phi'_{out}} \quad (38)$$

$$E_{+}^{a} = t_{out} \cdot \sqrt{T_a(+)} \cdot E_{+}^{w} \cdot e^{-i \cdot \phi'_{out}} \quad (39)$$

where $T_a(\parallel)$ and $T_a(+)$ denote analyzer's transmission for the light polarised linearly in parallel and perpendicularly to its axis, respectively. t_{out} describes an ant put transmission value being a product of amplitudes transmission through borders: glass-analyzer and analyzer-air (in the case of lack of on antireflective layer) and antireflective layer-air. Φ'_{out} denotes a phase shift for one of the disused cases.

Light entering a LC display from an observer side (external source of light) can be considered in the same way, but in this case interference effects have to be taken into account.

COMPUTER PROGRAMME CAPABILITIES

The computer programme we're worked out basing the above outlined mathematical considerations makes it possible to determine:

- luminance of on- and off-state;
- contrast ratio coefficient;
- chromatic coordinates;
- spectral characteristics of transmission;

If should be emphasised that there were taken into account dispersions of all parameters, human eyes spectral characteristics properties of external source of light, compound character of refractive indices, dichroism of a LC layer etc, so the obtained results approximate the real ones.

Experimental verification we're carried out confirmed agreement of theoretical and practical results at the level of 97%. Beneath there are presented exemplary results we obtained using the programme.

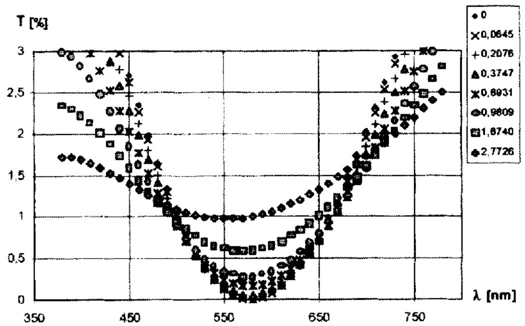

FIGURE 3 Exemplary characteristics of TN display working in transmission regime within the range of the 1st minimum of negative mode. Dependencies $T=f(\lambda)$ have been obtained for different values of layer dichroism. Reflections and external source of light have been neglected. The descriptions present values $d(\alpha_{||} - \alpha_{+})$.

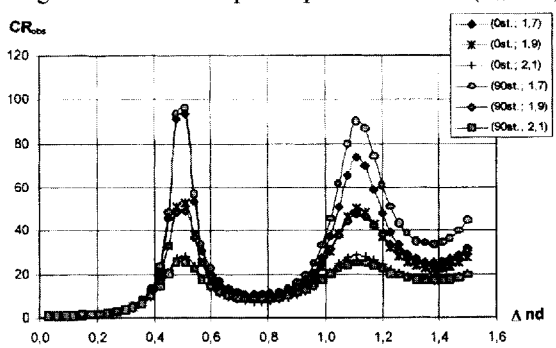

FIGURE 4 The course of function $CR = f(\Delta nd)$ (for different values of refractive index of conductive layer) in transmission regime of TN effect. The angle between rubbing direction and polarises axis from an observer side equals to 0^{o}. External luminance $I_{out}=0,5I_{in}$. Ideal polarizers.

The programme enables determining values of CR and luminance practically for any possible conditions for a standard construction of a LC display (without colour filters, compensating components etc.). Those elements can be included into the programme using by modification of the assumed mathematical model. This is not any restriction of the presented method, but it simply hasn't been considered in this paper.

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